



JOM SKOR A+

ADD MATH

DIFFERENTIATION



$$\begin{aligned} B^3 &= CD + DA \\ B^3 &= (D - C \sin B) \\ B^3 &= D^2 - 3A \cos B^2 + A \sin B \\ B^3 &= D^2 - 4A \cos B^3 + C \sin B \\ B^3 &= C^3 - A^2 - 3 \cos B \end{aligned}$$

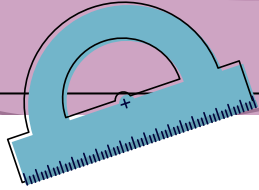


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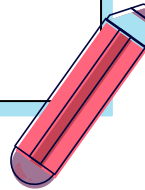
$$x_2^4 + x_3^2 = (x_2 + x_3)^2$$





Beauty in mathematics is
seeing the truth without effort

-GEORGE PÓLYA-



DOKUMEN STANDARD KURIKULUM DAN PENTAKSIRAN (DSKP)

PACKAGE	LEARNING AREA	TOPICS IN FORM FOUR	TOPICS IN FORM FIVE
Core	Algebra	<ul style="list-style-type: none"> • Functions • Quadratic Functions • Systems of Equations • Indices, Surds and Logarithms • Progressions • Linear Law 	
	Geometry	<ul style="list-style-type: none"> • Coordinate Geometry • Vectors 	<ul style="list-style-type: none"> • Circular Measure
	Calculus		<ul style="list-style-type: none"> • Differentiation • Integration
	Trigonometry		<ul style="list-style-type: none"> • Trigonometric Functions
	Statistics		<ul style="list-style-type: none"> • Permutation and Combination • Probability Distribution
Elective	Trigonometry (Application of Science and Technology)	<ul style="list-style-type: none"> • Solution of Triangles 	
	Statistics (Application of Social Science)	<ul style="list-style-type: none"> • Index Numbers 	
	Calculus (Application of Science and Technology)		<ul style="list-style-type: none"> • Kinematics of Linear Motion
	Algebra (Application of Social Science)		<ul style="list-style-type: none"> • Linear Programming



Past Years SPM

		2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Paper 1		15 16	20 21	19 20	17 18 19	19 20	19	19 20	20 21	20	19 20	19 20	17 18	16 23	6 7	1	5 6	17
Paper 2	Sec A	3	5	2		4		3			3	5		2	6	2	6	4
	Sec B	9	10	8	8	10	7	7	8	8	8	8	8	8	8	11	10	7

Prior Knowledge

- The Straight Line (Form 3)

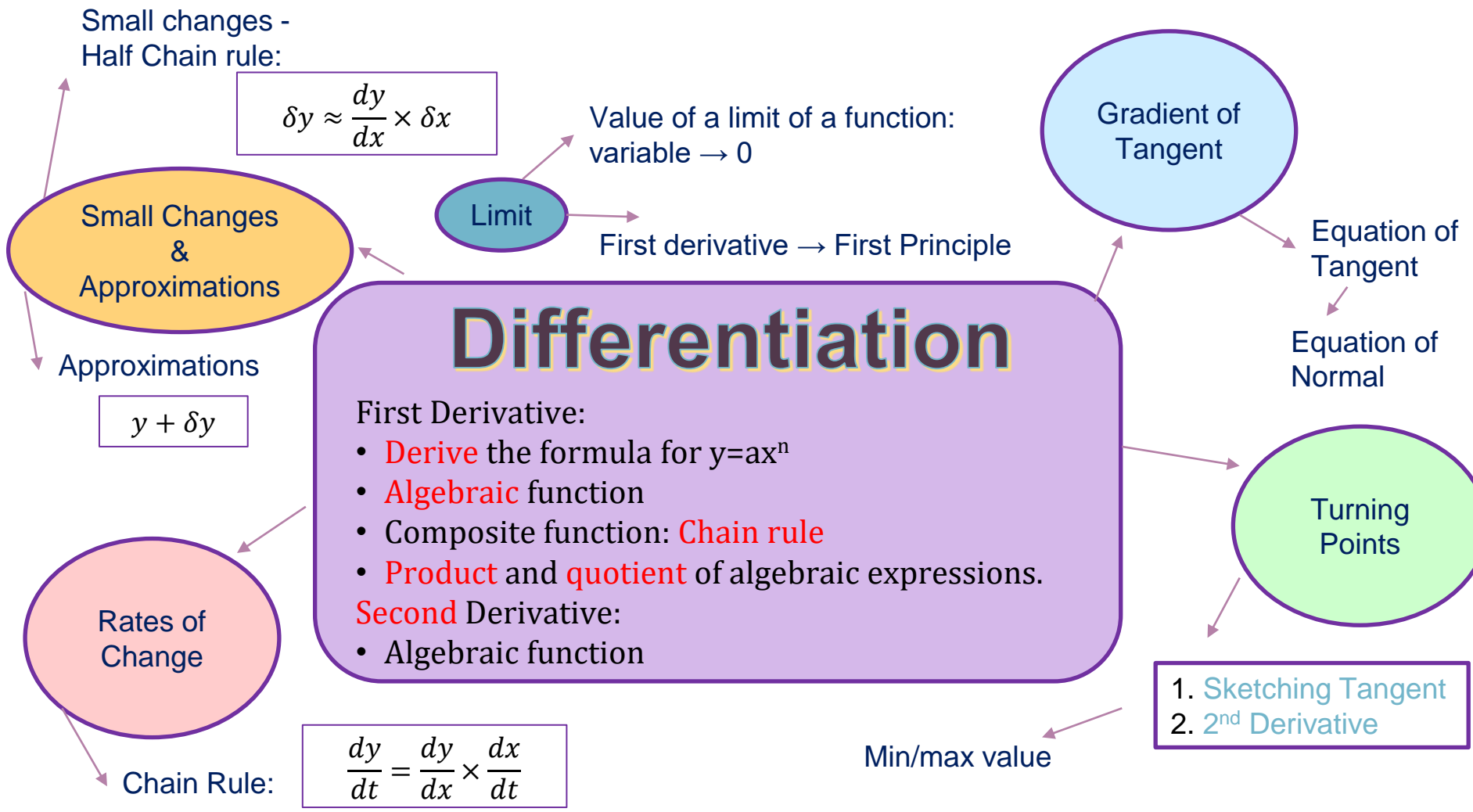
Equation of straight line
 $y = mx + c$

↓ ↓
Gradient y-intercept

- Angles and Tangents of Circles (Form 3)
- Coordinate Geometry (Add Math Form 4)

Two straight lines, L_1 and L_2 are perpendicular to each other if and only if $m_1 m_2 = -1$.

- Quadratic Functions (Add Math Form 4)
- Speed and Acceleration (Form 2)



Small changes -
Half Chain rule:

$$\delta y \approx \frac{dy}{dx} \times \delta x$$

Small Changes &
Approximations

Approximations

$$y + \delta y$$

Rates of
Change

Chain Rule:

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

Value of a limit of a function:
variable $\rightarrow 0$

Limit

First derivative \rightarrow First Principle

Differentiation

First Derivative:

- Derive the formula for $y=ax^n$
- Algebraic function
- Composite function: Chain rule
- Product and quotient of algebraic expressions.

Second Derivative:

- Algebraic function

Gradient of
Tangent

Equation of
Tangent

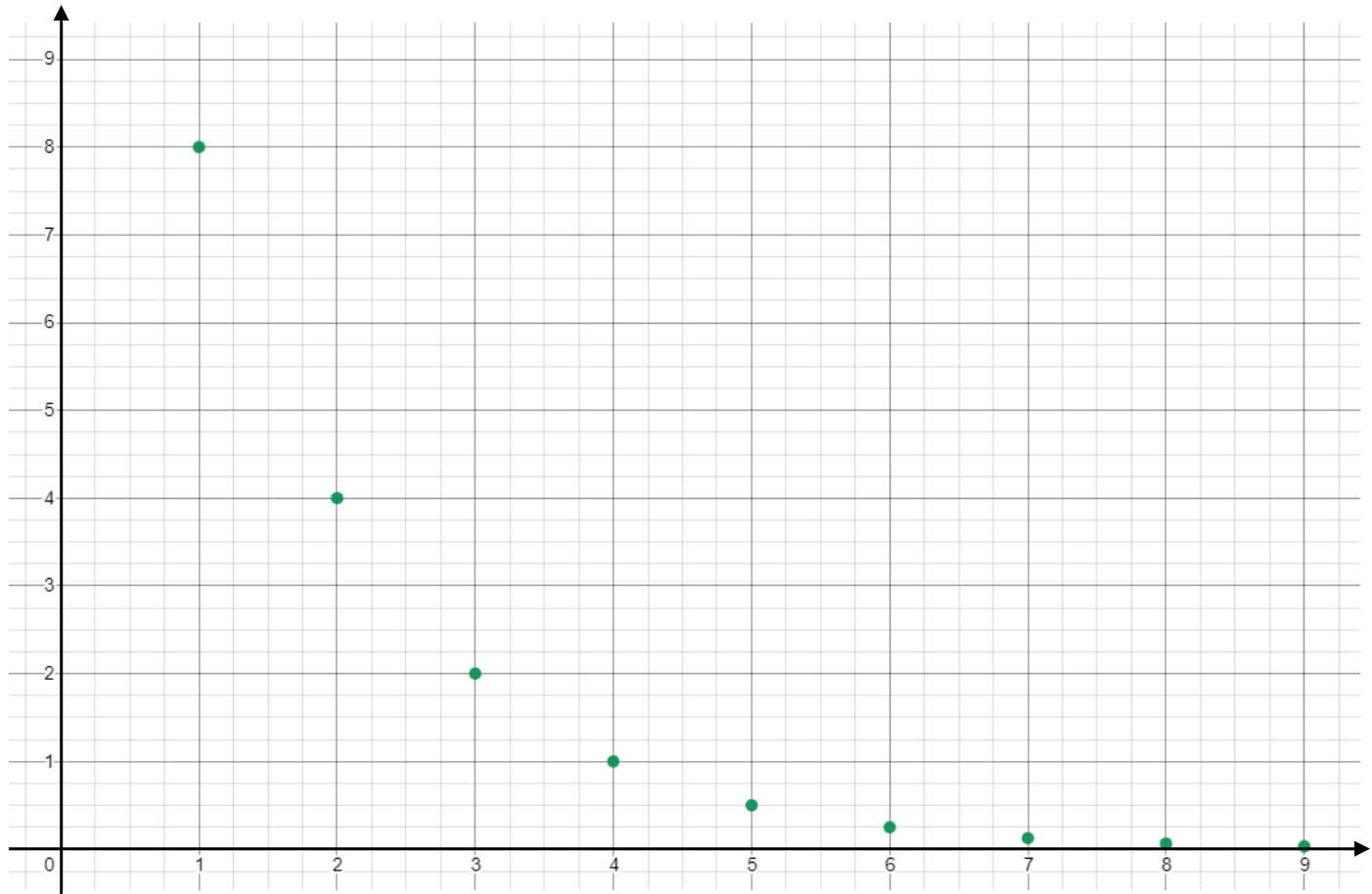
Equation of
Normal

Turning
Points

1. Sketching Tangent
2. 2nd Derivative

Min/max value

Limit



Example 1(a)

Determine the limit value for each of the following functions.

(a) $\lim_{x \rightarrow 0} \frac{1}{x}$

x	1	0.1	0.01	0.001	0.0001	0.00001	0.000001	0.0000001	...
$\frac{1}{x}$	1	10	100	1000	10000	100000	1000000	10000000	?

$$\lim_{x \rightarrow 0} \frac{1}{x} = \infty$$

Example 1(b)

Determine the limit value for each of the following functions.

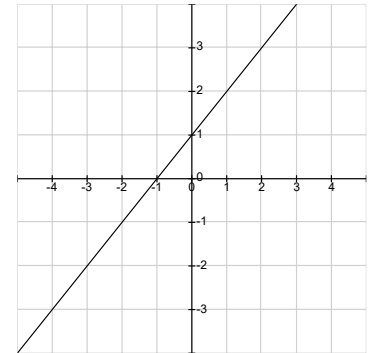
$$(a) \lim_{x \rightarrow 0} \frac{x^3 - x}{x^2 - x}$$

$$= \lim_{x \rightarrow 0} \frac{x(x^2 - 1)}{x(x - 1)}$$

$$= \lim_{x \rightarrow 0} \frac{x(x - 1)(x + 1)}{x(x - 1)}$$

$$= \lim_{x \rightarrow 0} x + 1$$

$$= 1$$



2.1.1 Investigate and determine the value of limit of a function when its variable approaches zero.



What is $\frac{dy}{dx}$?

Power turun
ke depan

Gradient
tangent

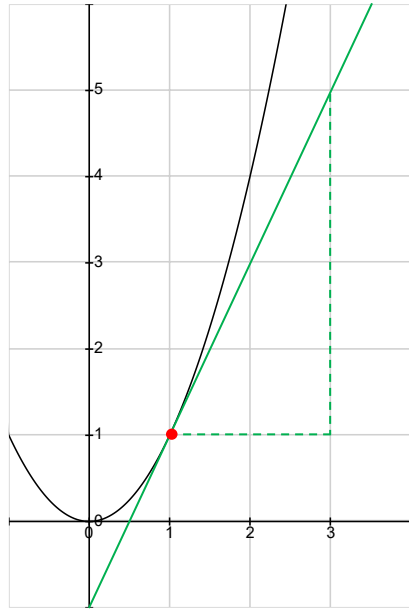
Rate of
something

Maximum
minimum

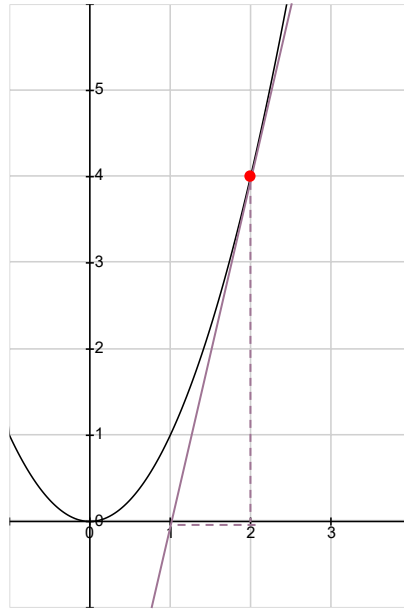
Reduce power



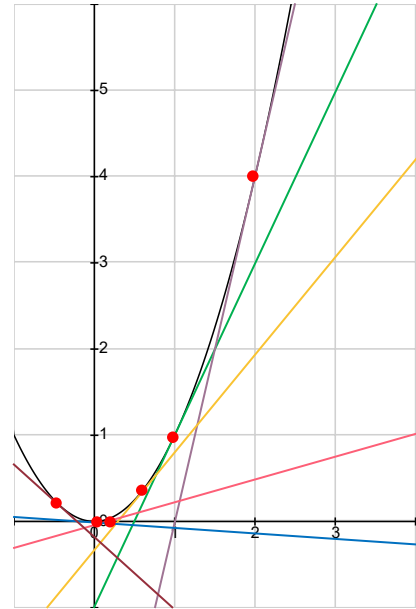
The gradient of a tangent...



$$m_t = \frac{5 - 1}{3 - 1} = 2$$

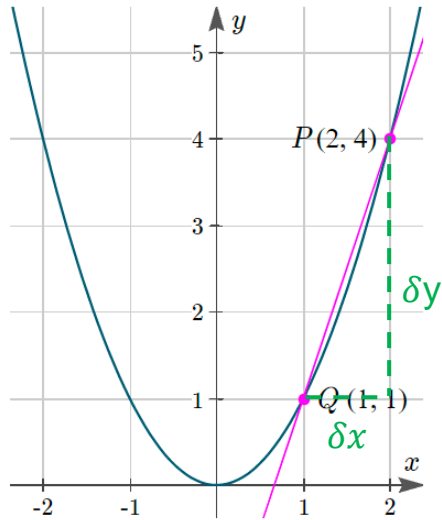


$$m_t = \frac{4 - 0}{2 - 1} = 4$$

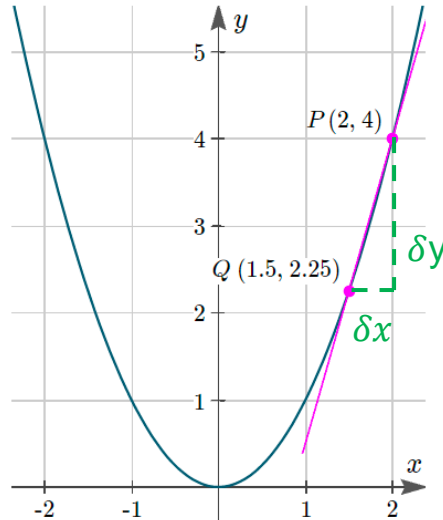


$m_t = ?$

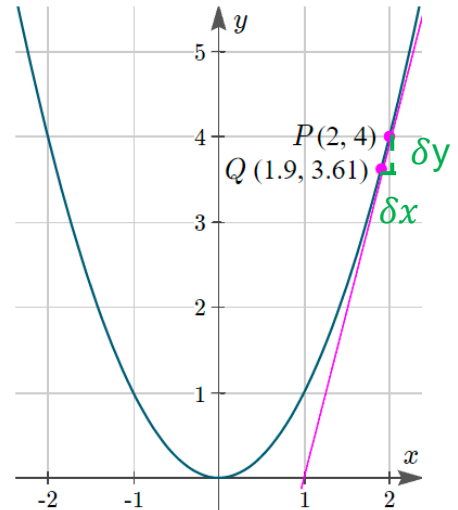
Idea of limit to differentiation...



$$m_{PQ} = \frac{4 - 1}{2 - 1} = \frac{3}{1} = 3$$

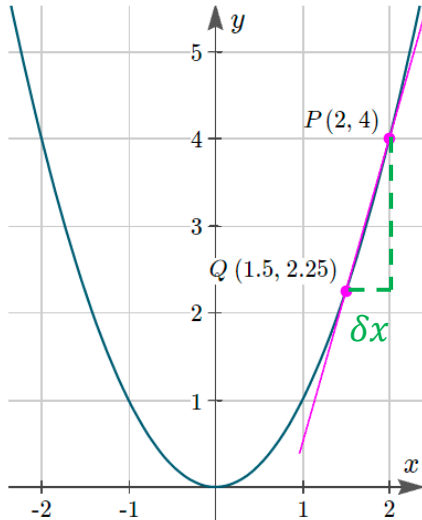


$$m_{PQ} = \frac{4 - 2.25}{2 - 1.5} = \frac{1.75}{0.5} = 3.5$$



$$m_{PQ} = \frac{4 - 3.61}{2 - 1.9} = \frac{0.39}{0.1} = 3.9$$

First derivative using first principle...



Gradient of tangent at $P = \text{Gradient of } PQ \text{ as } \delta x \text{ approaching } 0$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} \frac{f(x + \delta x) - f(x)}{\delta x}$$

Example 2(a)

First derivative using
first principle

Find the first derivative by using the first principles for each of the following functions $y = f(x)$

(a) $y = 2x$

Given $y = f(x) = 2x$

$$\delta y = f(x + \delta x) - f(x)$$

$$\delta y = 2(x + \delta x) - 2x$$

$$\delta y = 2x + 2\delta x - 2x$$

$$\delta y = 2\delta x$$

$$\frac{\delta y}{\delta x} = 2$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} 2$$

$$\frac{dy}{dx} = 2$$

Example 2(b)

First derivative using
first principle

Find the first derivative by using the first principles for each of the following functions $y = f(x)$

(b) $y = 2x^2 + 1$

Given $y = f(x) = 2x^2 + 1$

$$\delta y = f(x + \delta x) - f(x)$$

$$\delta y = [2(x + \delta x)^2 + 1] - [2x^2 + 1]$$

$$\delta y = [2(x^2 + 2x\delta x + (\delta x)^2) + 1] - [2x^2 + 1]$$

$$\delta y = 2x^2 + 4x\delta x + 2(\delta x)^2 + 1 - 2x^2 - 1$$

$$\delta y = 4x\delta x + 2(\delta x)^2$$

$$\delta y = \delta x(4x + 2\delta x)$$

$$\frac{\delta y}{\delta x} = 4x + 2\delta x$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} 4x + 2\delta x$$

$$\frac{dy}{dx} = 4x$$

Derive by induction

Function	$\frac{dy}{dx}$	Pattern
$y = 2x$	$\frac{dy}{dx} = 2$	$\frac{dy}{dx} = 2 \times 1 \times x^{1-1}$
$y = 2x^2$	$\frac{dy}{dx} = 4x$	$\frac{dy}{dx} = 2 \times 2 \times x^{2-1}$
$y = 2x^3$	$\frac{dy}{dx} = 6x^2$	$\frac{dy}{dx} = 2 \times 3 \times x^{3-1}$
$y = 2x^4$	$\frac{dy}{dx} = 8x^3$	$\frac{dy}{dx} = 2 \times 4 \times x^{4-1}$

Conclusions:

$$\text{If } y = ax^n, \text{ then } \frac{dy}{dx} = anx^{n-1}$$

2.2.1 Derive the formula of first derivative inductively for the function $y = ax^n$, a is a constant and n is an integer.

Example 3

First derivative of an algebraic function

$$y = ax^n \rightarrow \frac{dy}{dx} = anx^{n-1}$$

Find the first derivative for each of the following with respect to x

(a) $y = 3x^{10}$

$$\frac{dy}{dx} = 3(10)x^{10-1}$$

$$\frac{dy}{dx} = 30x^9$$

(b) $f(x) = x^3 + x^{-2} + 1$

$$f'(x) = 3x^{3-1} + (-2)x^{-2-1} + 1(0)x^{0-1}$$

$$f'(x) = 3x^2 - 2x^{-3}$$

(c) $\frac{1 - 2x^3}{x^2}$

$$\frac{d}{dx} \left(\frac{1 - 2x^3}{x^2} \right) = \frac{d}{dx} \left(\frac{1}{x^2} - \frac{2x^3}{x^2} \right)$$

$$= \frac{d}{dx} (x^{-2} - 2x)$$

$$= (-2)x^{-2-1} - 2(1)x^{1-1}$$

$$= -2x^{-3} - 2$$

$$= -\frac{2}{x^3} - 2 = -\frac{2 + 2x^3}{x^3}$$

Example 4

First derivative of composite function (chain rule)

Find the first derivative of the following with respect to x : $y = (2x + 1)^5$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Let $u = 2x + 1$ then $y = u^5$

$$\frac{du}{dx} = 2 \qquad \frac{dy}{du} = 5u^4$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = 5u^4 \times 2$$

$$\frac{dy}{dx} = 10u^4$$

$$\frac{dy}{dx} = 10(2x + 1)^4$$

Simplified

$$y = (2x + 1)^5$$

$$\frac{dy}{dx} = 5(2x + 1)^{5-1}(2)$$

$$\frac{dy}{dx} = 10(2x + 1)^4$$

Example 5

First derivative of a function involving product rule

Find the first derivative of the following with respect to x : $y = (1 - 2x)(x^2 - 2)^3$

$$\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$\text{Let } y = uv \quad u = 1 - 2x \quad v = (x^2 - 2)^3$$

$$\frac{du}{dx} = -2 \quad \frac{dv}{dx} = 3(x^2 - 2)^2(2x) = 6x(x^2 - 2)^2$$

$$\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$\frac{dy}{dx} = (x^2 - 2)^3(-2) + (1 - 2x)[6x(x^2 - 2)^2]$$

$$\frac{dy}{dx} = 2(x^2 - 2)^2[(x^2 - 2)(-1) + (1 - 2x)(3x)]$$

$$\frac{dy}{dx} = 2(x^2 - 2)^2(2 + 3x - 7x^2)$$

Example 6

First derivative of a function involving quotient rule

Find the first derivative of the following with respect to x : $y = \frac{x^2}{3 - 4x}$

$$\text{Let } y = \frac{u}{v}$$

$$u = x^2 \qquad v = 3 - 4x$$

$$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = -4$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{(3 - 4x)(2x) - x^2(-4)}{(3 - 4x)^2}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{6x - 8x^2 + 4x^2}{(3 - 4x)^2}$$

$$\frac{dy}{dx} = \frac{6x - 4x^2}{(3 - 4x)^2}$$

$$\frac{dy}{dx} = \frac{2x(3 - 2x)}{(3 - 4x)^2}$$

Example 7

Second derivative

Find the second derivative of the following with respect to x : $y = x^2 + 1$

$$\frac{dy}{dx} = 2x$$

$$\frac{d^2y}{dx^2} = 2$$

Example 8

Second derivative

Given $y = x^2 + 2x + 7$, find the values of x if $x^2 \frac{d^2y}{dx^2} + (x - 1) \frac{dy}{dx} + y = 8$

$$y = x^2 + 2x + 7$$

$$\frac{dy}{dx} = 2x + 2$$

$$\frac{d^2y}{dx^2} = 2$$

$$x^2(2) + (x - 1)(2x + 2) + x^2 + 2x + 7 = 8$$

$$2x^2 + 2x^2 - 2 + x^2 + 2x + 7 - 8 = 0$$

$$5x^2 + 2x - 3 = 0$$

$$(5x - 3)(x + 1) = 0$$

$$x = \frac{3}{5} \quad x = -1$$

SPM 2005 Q19

Given that $h(x) = \frac{1}{(3x - 5)^2}$, evaluate $h''(1)$

$$h(x) = (3x - 5)^{-2}$$

$$h'(x) = (-2)(3x - 5)^{-2-1}(3)$$

$$h'(x) = -6(3x - 5)^{-3}$$

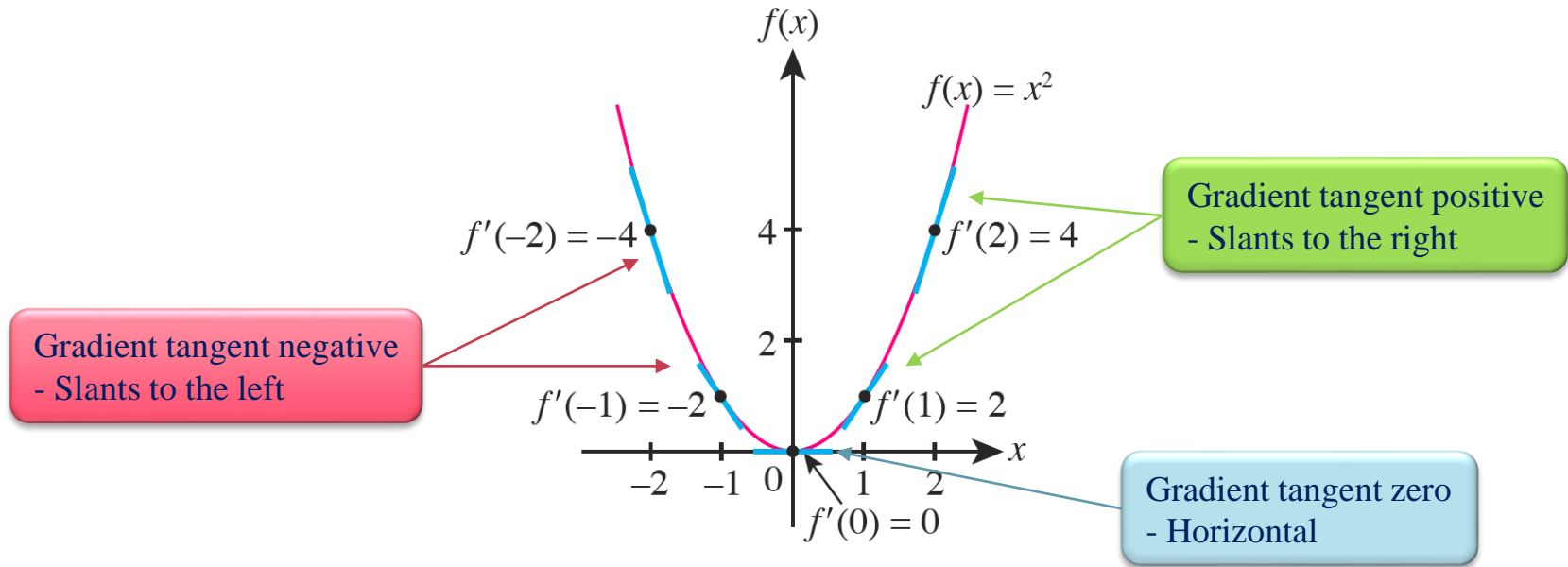
$$h''(x) = (-3)(-6)(3x - 5)^{-3-1}(3)$$

$$h''(x) = 54(3x - 5)^{-4}$$

$$h''(1) = 54(3(1) - 5)^{-4}$$

$$h''(1) = \frac{27}{8}$$

Gradient of tangent to a curve



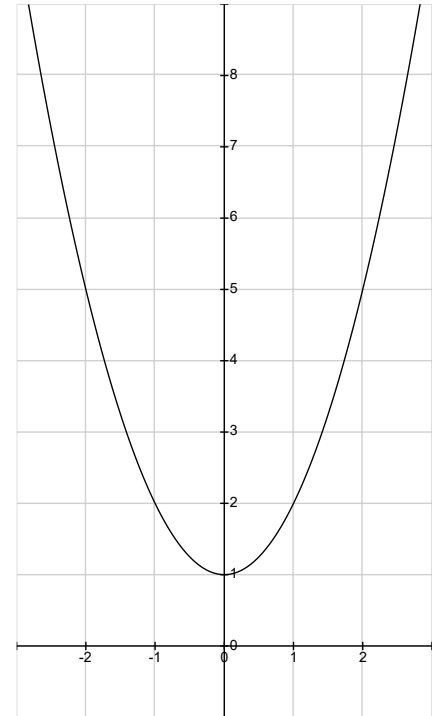
Example 9(a)

Application: Interpret
gradient of tangent

The diagram on the right shows a part of the curve $y = x^2 + 1$

(a) Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = 2x$$



Example 9(b)(c)

Application: Interpret gradient of tangent

The diagram on the right shows a part of the curve $y = x^2 + 1$

(b) Find the gradient of the tangent to the curve at points A , B and C .

(c) For each of the points, state the condition of the tangent.

$$\frac{dy}{dx} = 2x \qquad \frac{dy}{dx} = m_t$$

$$A(-1,2)$$

$$B(0,1)$$

$$C(2,5)$$

$$m_t = 2(-1)$$

$$m_t = 2(0)$$

$$m_t = 2(2)$$

$$m_t = -2$$

$$m_t = 0$$

$$m_t = 4$$

$$m_t < 0$$

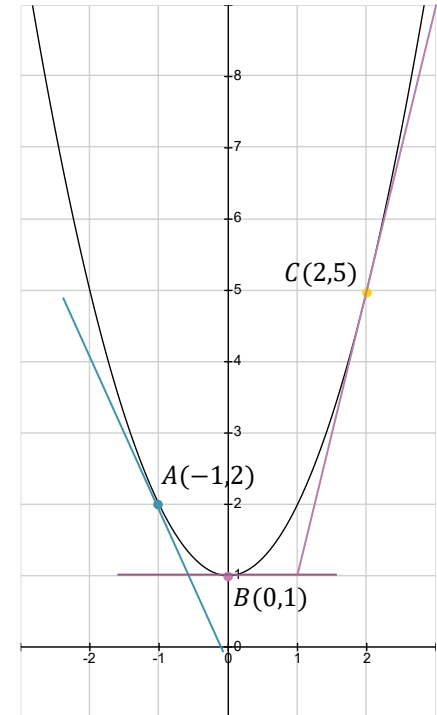
$$m_t = 0$$

$$m_t > 0$$

negative – slant to left

zero – horizontal

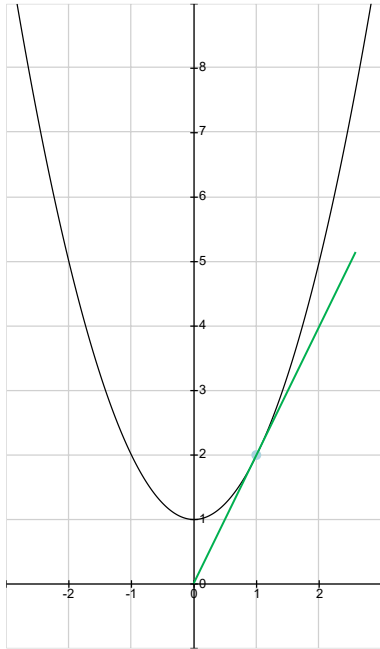
positive – slants to right



Example 10

Application: Equation of
tangent and normal

Find the equation of the tangent and normal to the curve $y = x^2 + 1$ at point $R(1,2)$



$$\frac{dy}{dx} = 2x$$

$$\text{when } x = 1$$

$$\frac{dy}{dx} = m_t = 2(1)$$

$$m_t = 2$$

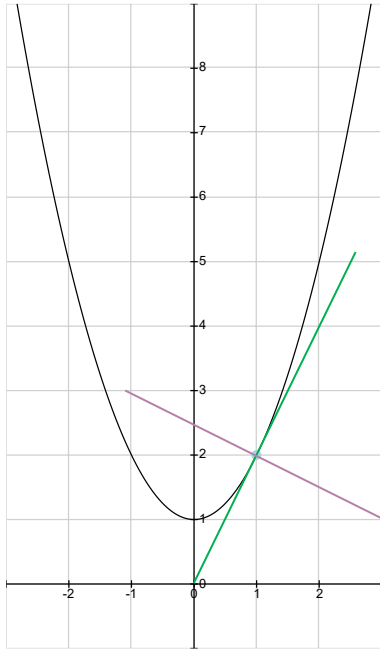
$$y - 2 = 2(x - 1)$$

$$y = 2x \quad \leftarrow \text{Equation of tangent}$$

Example 10

Application: Equation of tangent and normal

Find the equation of the tangent and normal to the curve $y = x^2 + 1$ at point $R(1,2)$



$$m_t = 2$$

$$m_t \times m_n = -1$$

$$2 \times m_n = -1$$

$$m_n = -\frac{1}{2}$$

$$y - 2 = -\frac{1}{2}(x - 1)$$

$$y = -\frac{1}{2}x + \frac{5}{2}$$

$$2y = 5 - x \quad \longleftarrow \text{Equation of normal}$$

SPM 2006 Q17

The point P lies on the curve $y = (x - 5)^2$. It is given that the gradient of the normal at P is $-\frac{1}{4}$.
Find the coordinates of P

$$y = (x - 5)^2 \qquad m_n = -\frac{1}{4}$$

$$\frac{dy}{dx} = 2(x - 5)^{2-1}(1) \qquad m_t \times -\frac{1}{4} = -1$$

$$\frac{dy}{dx} = 2x - 10 \qquad m_t \times m_n = -1$$

$$m_t = 4$$

$$2x - 10 = 4$$

$$2x = 4 + 10$$

$$x = \frac{14}{2}$$

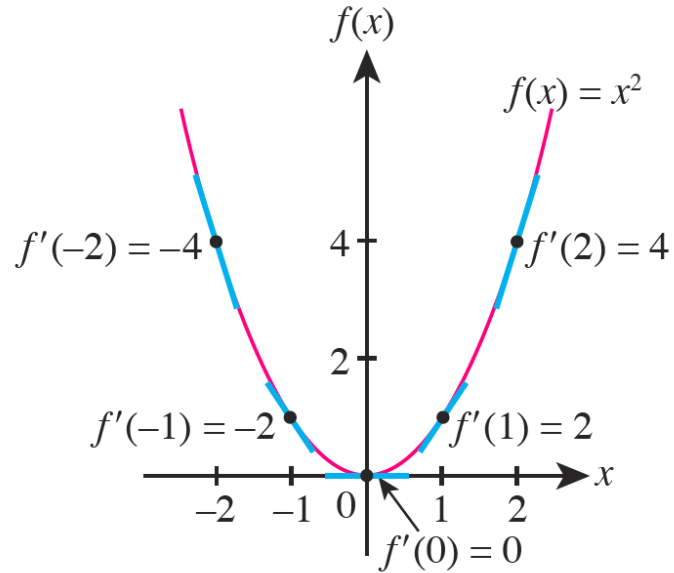
$$x = 7$$

$$y = ((7) - 5)^2$$

$$y = 4$$

$$P(7,4)$$

Turning points



Example 11(a)

Application: Turning points

Given the curve: $y = x^3 - 9x^2 + 24x - 15$

(a) Find the coordinates of turning points of the curve.

$$\frac{dy}{dx} = 3x^2 - 18x + 24$$

$$\frac{dy}{dx} = 0$$

$$3x^2 - 18x + 24 = 0$$

$$3(x - 2)(x - 4) = 0$$

$$x - 2 = 0$$

$$x = 2$$

$$y = (2)^3 - 9(2)^2 + 24(2) - 15$$

$$y = 5$$

(2,5)

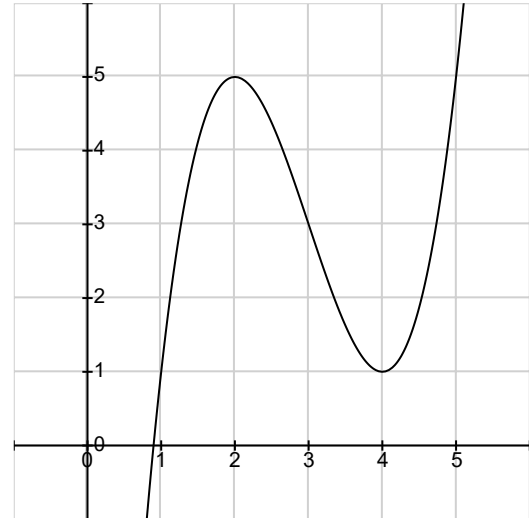
$$x - 4 = 0$$

$$x = 4$$

$$y = (4)^3 - 9(4)^2 + 24(4) - 15$$

$$y = 1$$

(4,1)










Example 11(b)

Application: Turning points

Given the curve: $y = x^3 - 9x^2 + 24x - 15$

(b) Determine whether each of the turning points is a maximum or minimum point.

$$\frac{dy}{dx} = 3x^2 - 18x + 24 \quad (2,5) \quad (4,1)$$

x	1	2	3	4	5
$\frac{dy}{dx}$	9	0	-3	0	9
Sign of the tangent	+	0	-	0	+
Sketch of the tangent					
Sketch of the graph					

(2,5) is a maximum point (4,1) is a minimum point

$$\frac{d^2y}{dx^2} = 6x - 18$$

$$\text{For } (2,5) \quad \frac{d^2y}{dx^2} = 6(2) - 18 = -4 < 0$$

(2,5) is a maximum point

$$\text{For } (4,1) \quad \frac{d^2y}{dx^2} = 6(4) - 18 = 4 > 0$$

(4,1) is a minimum point

Example 12

Application: Turning points

Given the curve: $y = x^3 + 1$

Find the stationary points and determine the nature of it.

$$\frac{dy}{dx} = 3x^2$$




$$3x^2 = 0$$

$$x = 0$$

$$y = (0)^3 + 1$$

$$y = 1$$

$$(0,1)$$

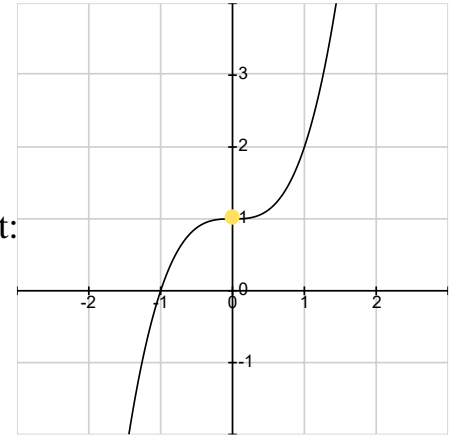
x	-1	0	1
$\frac{dy}{dx}$	3	0	3
Sign of the tangent	+	0	+
Sketch of the tangent			
Sketch of the graph			

$$\frac{d^2y}{dx^2} = 6x$$

Stationary point:
(0,1)

$$\frac{d^2y}{dx^2} = 6(0)$$

$$\frac{d^2y}{dx^2} = 0$$



Point of Inflection

SPM 2003 Q15

Application: Maximum and minimum values

Given that $y = 14x(5 - x)$, calculate

- (a) the value of x when y is maximum,
- (b) the maximum value of y . [3 marks]

$$y = 70x - 14x^2$$

$$\frac{dy}{dx} = 70 - 28x$$

$$70 - 28x = 0$$

$$70 = 28x$$

$$x = \frac{5}{2}$$

$$y = 70\left(\frac{5}{2}\right) - 14\left(\frac{5}{2}\right)^2$$

$$y = \frac{175}{2}$$

Example 13

Application: Rate of change

A curve has an equation $y = x^2 - 4x + 6$. Find the rate of change of y when $x = 1$ and $x = 4$, given that x increases at a constant rate of 2 units per second.

$$\frac{dy}{dt} = ? \quad \frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$\frac{dy}{dx} = 2x - 4$$

$$\frac{dy}{dt} = (2x - 4) \times 2$$

$$\frac{dy}{dt} = 4x - 8$$

when $x = 1$

$$\frac{dy}{dt} = 4(1) - 8$$

$$\frac{dy}{dt} = -4$$

y decreases at rate
of 4 unit per second

when $x = 4$

$$\frac{dy}{dt} = 4(4) - 8$$

$$\frac{dy}{dt} = 8$$

y increases at rate
of 8 unit per second

SPM 2004 Q21

Two variables, x and y are related by the equation $y = 3x + \frac{2}{x}$. Given that y increases at a constant rate of 4 units per second, find the rate of change of x when $x = 2$

$$\frac{dy}{dt} = 4$$

$$\frac{dx}{dt} = ? \text{ when } x = 2$$

$$y = 3x + \frac{2}{x}$$

$$y = 3x + 2x^{-1}$$

$$\frac{dy}{dx} = 3 + 2(-1)x^{-1-1}$$

$$\frac{dy}{dx} = \frac{3x^2 - 2}{x^2}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$4 = \frac{3x^2 - 2}{x^2} \times \frac{dx}{dt}$$

$$4 = \frac{3(2)^2 - 2}{(2)^2} \times \frac{dx}{dt}$$

$$4 \times \frac{4}{10} = \frac{dx}{dt}$$

$$\frac{dx}{dt} = 1.6$$

Example 14

Application: Small Changes

Find the small corresponding change in $y = x^2 + 1$, when x increases from 1 to 1.02

$$\delta x = 1.02 - 1$$

$$\delta x = 0.02$$

$$\delta y = ?$$

$$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$$

$$\delta y = \frac{dy}{dx} \times \delta x \quad \frac{dy}{dx} = 2x$$

$$\delta y = 2x \times 0.02$$

$$\delta y = 0.04x$$

$$\delta y = 0.04(1)$$

$$\delta y = 0.04$$

SPM 2006 Q19

Given that $y = 3x^2 + x - 4$

- (a) Find the value of $\frac{dy}{dx}$ when $x = 1$
- (b) express the approximate change in y , in terms of p , when x changes from 1 to $1 + p$, where p is a small value.

$$\frac{dy}{dx} = 6x + 1$$

$$\text{when } x = 1, \frac{dy}{dx} = 6(1) + 1$$

$$\frac{dy}{dx} = 7$$

$$\delta x = (1 + p) - 1$$

$$\delta x = p$$

$$\delta y \approx \frac{dy}{dx} \times \delta x$$

$$\delta y = 7 \times p$$

$$\delta y = 7p$$

Example 15

Application: Approximation

Given $y = \frac{16}{x^2}$, find the value of $\frac{dy}{dx}$ when $x = 2$ and determine the approximate value for $\frac{16}{2.01^2}$.

$$y = 16x^{-2}$$

$$\frac{dy}{dx} = -32x^{-3}$$

$$\frac{dy}{dx} = -\frac{32}{x^3}$$

$$\text{when } x = 2$$

$$\frac{dy}{dx} = -\frac{32}{(2)^3}$$

$$\frac{dy}{dx} = -4$$

$$\delta x = 2.01 - 2$$

$$\delta x = 0.01$$

$$\delta y \approx \frac{dy}{dx} \times \delta x$$

$$\delta y = -4 \times 0.01$$

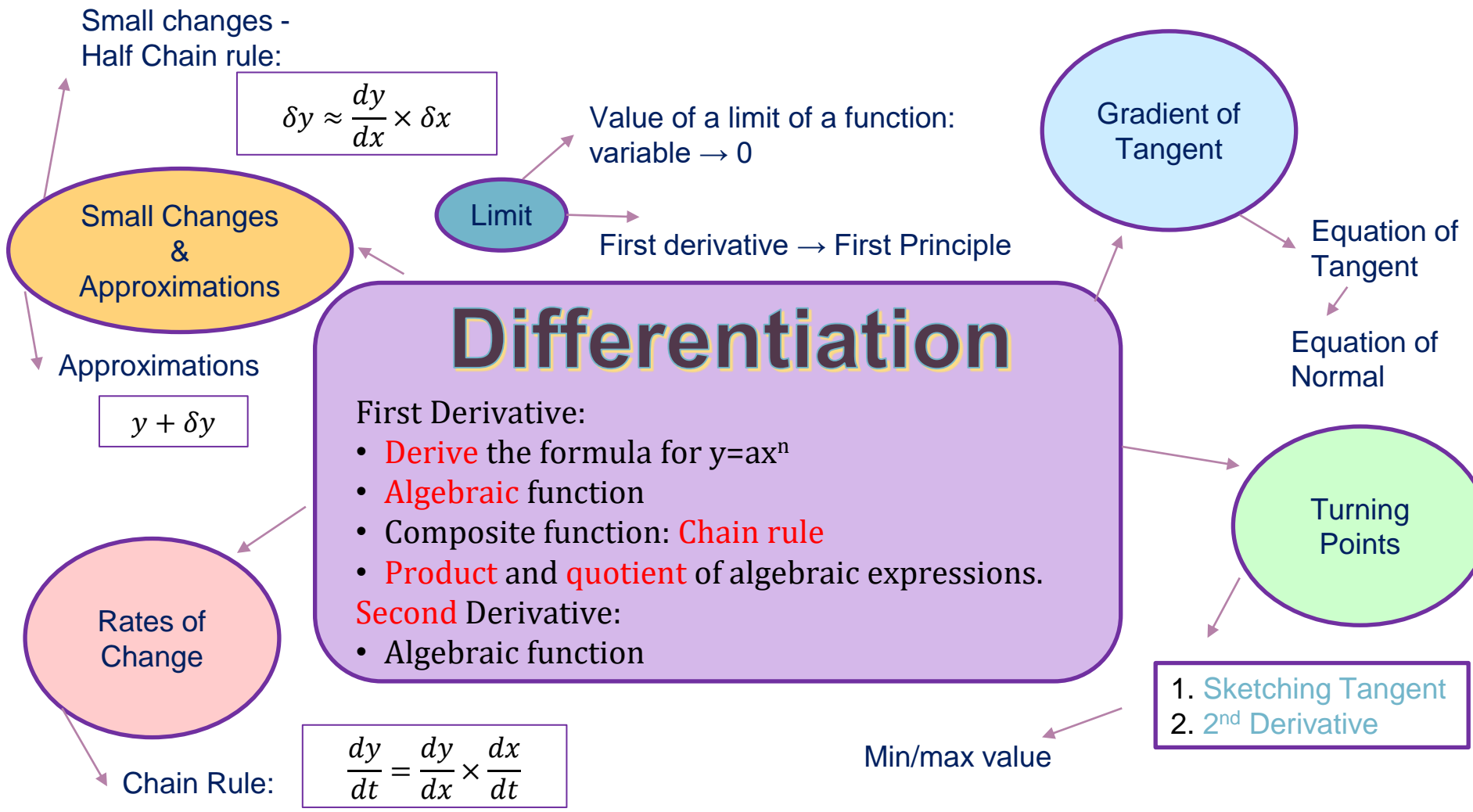
$$\delta y = -0.04$$

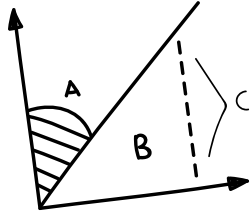
$$\frac{16}{2.01^2} \approx y + \delta y$$

$$= \frac{16}{(2)^2} + (-0.04)$$

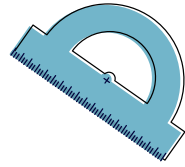
$$= 4 - 0.04$$

$$= 3.96$$

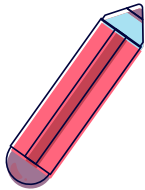




01

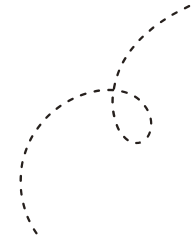


$$(-3\sqrt{2}) - 4(3)(-3M+2)$$



PAST YEARS

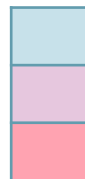
SPM 3472/1



$$\frac{3 \sin 4/8}{\sqrt{3 \cdot 2 \cdot 4 + 2}}$$

Past Years SPM

		2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Paper 1		15	20	19	17	19	19	19	20	20	19	19	17	16	6	1	5	17
		16	21	20	18	20		20	21		20	20	18	23	7		6	
		19	20	20	19	20	19	20	21	20	20	18	23	7	6			
Paper 2	Sec A	3	5	2		4		3			3	5		2	6	2	6	4
	Sec B	9	10	8	8	10	7	7	8	8	8	8	8	8	8	11	10	7



First & Second derivative

Tangent / Normal

Turning Points (max/min)



Rate of Change

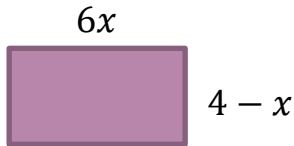
Approximation & Small Changes

Limit

SPM 2014 Q17

Due to the high living cost, Siva has planted several types of vegetables for his own consumption on a rectangular shape empty plot of land behind his house. He plans to fence the land which has a dimension of $6x$ m and $(4 - x)$ m.

Find the length, in m, the fence he has to buy when the area of the land is maximum. [4 marks]



perimeter = ?

$$\frac{dA}{dx} = 0$$

$$A = 6x \times (4 - x)$$

$$A = 24x - 6x^2$$

$$\frac{dA}{dx} = 24 - 12x$$

$$0 = 24 - 12x$$

$$x = 2$$

$$\text{perimeter} = 2(6x) + 2(4 - x)$$

$$= 10x + 8$$

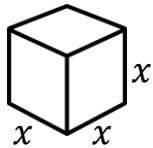
$$= 10(2) + 8$$

$$= 28$$

SPM 2016 Q6

The surface area of a cube increases at a constant rate of $15 \text{ cm}^2\text{s}^{-1}$

Find the rate of change of side length, in cm s^{-1} , when the volume of the cube is 125 cm^3 . [4 marks]



$$\frac{dA}{dt} = 15$$

$$\frac{dx}{dt} = ? \quad \text{when } V = 125$$

$$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$$

$$15 = 12x \times \frac{dx}{dt}$$

$$15 = 12(5) \times \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{4}$$

$$A = 6x^2$$
$$\frac{dA}{dx} = 12x$$

$$V = x^3$$
$$x^3 = 125$$

$$x = \sqrt[3]{125}$$
$$x = 5$$

SPM 2017 Q1

Diagram shows a cubic graph $y = f(x)$ and a linear graph $y = g(x)$.

Point A lies on the straight line. Points B , C and D lie on the curve. The tangents to the curve at points B and C are parallel to the x -axis.

State which point(s) that satisfies the following condition:

(a) $\frac{dy}{dx} = 0$

(b) $\frac{dy}{dx} < 0$

(c) $\frac{d^2y}{dx^2} > 0$

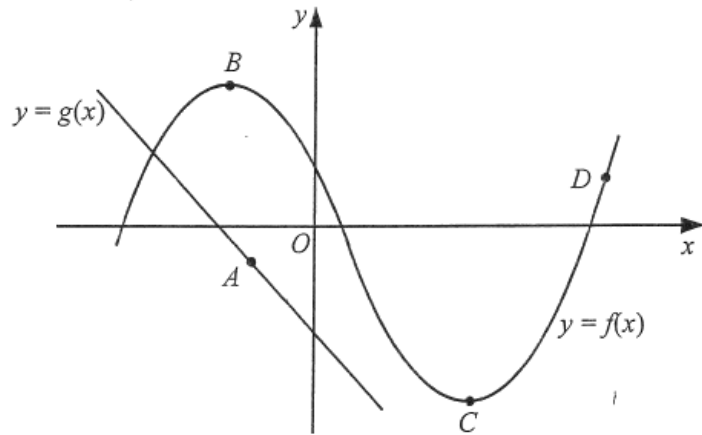


Diagram 1

SPM 2018 Q5

Find the value of


$$(a) \lim_{x \rightarrow 1} (7 - x^2) \qquad 6$$

$$(b) f''(2) \text{ if } f'(x) = 2x^3 - 4x + 3$$

$$f''(x) = 6x^2 - 4$$

$$f''(2) = 6(2)^2 - 4$$

$$f''(2) = 20$$

$$C = \sin^2\left(\frac{2}{3}\right)$$

$$= \sin^3 \times 0.747$$
$$= 7.38$$



$$A^3 C^2 4^B = 9^3 + 5^8 + 7^C$$
$$5^C = 54718.32.$$



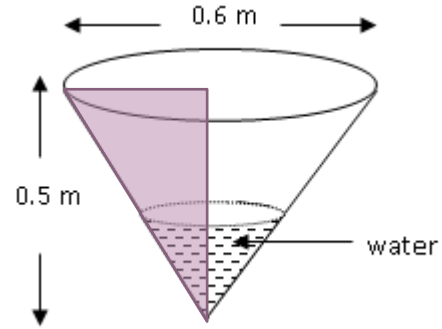
02

PAST YEARS SPM 3472/2

SPM 2003 Q9(a)

Diagram shows a conical container of diameter 0.6 m and height 0.5 m. Water is poured into the container at a constant rate of $0.2 \text{ m}^3 \text{ s}^{-1}$.

Calculate the rate of change of the height of the water level at the instant when the height of the water level is 0.4 m.



$$\frac{dV}{dt} = 0.2 \quad \frac{dh}{dt} = ? \quad \text{when } h = 0.4$$

[4 marks]

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

$$0.2 = \frac{9}{25} \pi h^2 \times \frac{dh}{dt}$$

$$0.2 = \frac{9}{25} \pi (0.4)^2 \times \frac{dh}{dt} \quad \frac{dh}{dt} = \frac{125}{36\pi}$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{3}{5} h \right)^2 h$$

$$V = \frac{3}{25} \pi h^3$$

$$\frac{dV}{dh} = \frac{9}{25} \pi h^2$$

$$r : h = 0.3 : 0.5$$

$$\frac{r}{h} = \frac{3}{5}$$

$$r = \frac{3}{5} h$$

SPM 2005 Q2(a)

A curve has a gradient function $px^2 - 4x$, where p is a constant. The tangent to the curve at the point $(1,3)$ is parallel to the straight line $y + x - 5 = 0$.

Find the value of p .

[3 marks]

$$y + x - 5 = 0$$

$$y = -x + 5$$

$$m_t = -1$$

$$\frac{dy}{dx} = px^2 - 4x$$

$$-1 = p(1)^2 - 4(1)$$

$$-1 + 4 = p$$

$$p = 3$$

SPM 2015 Q2

It is given the equation of the curve is $y = 2x(1 - x)^4$ and the curve passes through $P(2,4)$. Find

- a) the gradient of the curve at point P , [4 marks]

$$m_t = \frac{dy}{dx} = ? \quad \text{when } x = 2$$

$$\text{Let } y = uv$$

$$u = 2x$$

$$v = (1 - x)^4$$

$$\frac{du}{dx} = 2$$

$$\frac{dv}{dx} = 4(1 - x)^{4-1}(-1) = -4(1 - x)^3$$

$$\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$\frac{dy}{dx} = (1 - x)^4(2) + 2x[-4(1 - x)^3]$$

$$m_t = (1 - 2)^4(2) + 2(2)[-4(1 - 2)^3]$$

$$m_t = 18$$

SPM 2015 Q2

It is given the equation of the curve is $y = 2x(1 - x)^4$ and the curve passes through $P(2,4)$. Find

b) the equation of the normal to the curve at point P .

$$y = m_n x + c$$

$$m_t \times m_n = -1$$

$$18 \times m_n = -1$$

$$m_n = -\frac{1}{18}$$

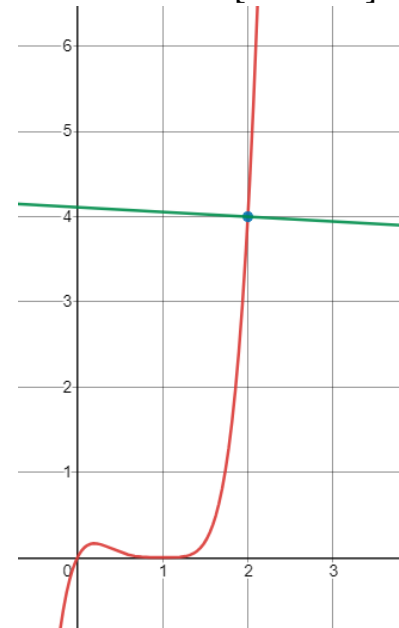
$$4 = -\frac{1}{18}(2) + c$$

$$c = \frac{37}{9}$$

$$y = -\frac{1}{18}x + \frac{37}{9}$$

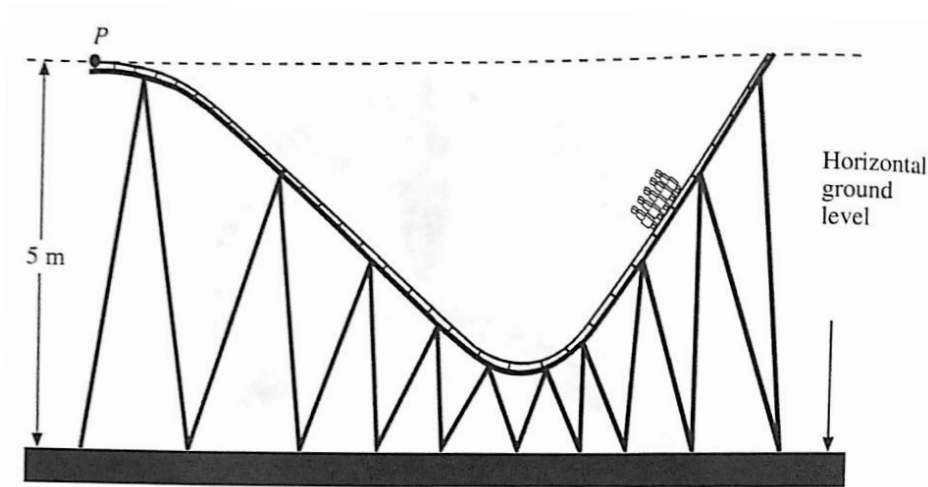
$$18y = 74 - x$$

[3 marks]



SPM 2018 Q6

Diagram shows the front view of a part of a roller coaster track in a miniature park.



The curve part of the track of the roller coaster is represented by an equation $y = \frac{1}{64}x^3 - \frac{3}{16}x^2$ with point P as the origin.

Find the shortest vertical distance, in m, from the track to the ground level.

[6 marks]

SPM 2018 Q6

$$y = \frac{1}{64}x^3 - \frac{3}{16}x^2 \quad \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{1}{64}(3)x^{3-1} - \frac{3}{16}(2)x^{2-1}$$

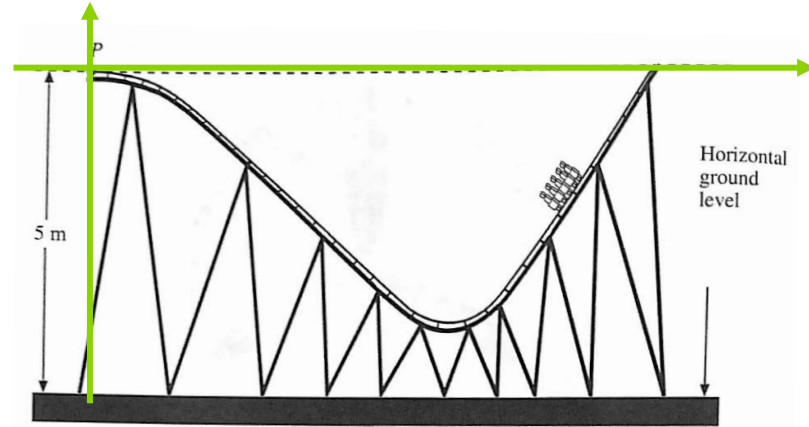
$$\frac{dy}{dx} = \frac{3}{64}x^2 - \frac{3}{8}x$$

$$\frac{3}{64}x^2 - \frac{3}{8}x = 0$$

$$x\left(\frac{3}{64}x - \frac{3}{8}\right) = 0$$

$$x = 0 \quad \frac{3}{64}x - \frac{3}{8} = 0$$

$$x = 8$$



$$y = \frac{1}{64}(8)^3 - \frac{3}{16}(8)^2$$

$$y = -4$$

$$\text{shortest vertical distance} = 5 - 4$$

$$= 1$$



Siri Jom Skor A+ Matematik Tambahan

SPM 2021

22 Aug 8.00 pm – 10.00 pm
Sahlawati Zakaria | MRSM Kuala Krai
Functions

27 Aug 8.00 pm – 10.00 pm
Norlela Sapari | MRSM Taiping
Quadratic Functions

31 Aug 8.00 pm – 10.00 pm
Khairulbariah Khairuddin | MRSM Mersing
Systems of Equations

4 Sept 8.00 pm – 10.00 pm
Hazlina Ahmad | MRSM Alor Gajah
Indices, Surds and Logarithms

10 Sept 8.00 pm – 10.00 pm
Hashiza Ismail | MRSM Parit
Progressions

16 Sept 3.00 pm – 5.00 pm
Rosdiana Sarju | MRSM Johor Bahru
Linear Law

24 Sept 8.00 pm – 10.00 pm
Nur Suhaifa Abu Bakar | MRSM Tumpat
Coordinate Geometry **New Speaker**

26 Sept 8.00 pm – 10.00 pm
Mohd Faizi Mamat | MRSM Gemencheh
Vectors

1 Oct 8.00 pm – 10.00 pm
Abdul Hadi Azmi | MRSM Pengkalan Chepa
Solution of Triangles

8 Oct 8.00 pm – 10.00 pm
Noraini Ismail | MRSM Transkrian
Index Numbers

10 Oct 8.00 pm – 10.00 pm
Hariani Abidin | MRSM Kuching
Circular Measure

15 Oct 8.00 pm – 10.00 pm
Erwan Hazreen Musa | MRSM Bentong
Differentiation

18 Oct 8.00 pm – 10.00 pm
Mohamad Fauzi Razak | MRSM Kepala Batas
Integration **New Date**

23 Oct 8.00 pm – 10.00 pm
Muhamad Baginda Zainuddin | MRSM Batu Pahat
Kinematics of Linear Motion **New Date**

30 Oct 3.00 pm – 5.00 pm
Haziq Syazwan Sajali | MRSM Tun Mustapha
Trigonometric Function **New Date**

5 Nov 3.00 pm – 5.00 pm
Suhaila Sulong | MRSM Tun Dr. Ismail
Permutation and Combination **New Date**

7 Nov 8.00 pm – 10.00 pm
Norhafizah Mohamed Yusoff | MRSM K. Terengganu
Probability Distribution **New Date**

17 Dis 8.00 pm – 10.00 pm
Asniza Arshad | MRSM Tun Ghaffar Baba
Linear Programming

Anjuran Unit Matematik
Bahagian Pendidikan Menengah MARA

Sesi webinar *live* melalui Microsoft Teams



Jom Skor A+ SPM 2021

MATEMATIK TAMBAHAN

Integration

18 October 2021 | Monday

8.00 pm – 10.00 pm

<https://bit.ly/CgFauziIntegrations>



Cg. Mohamad Fauzi Razak
MRSM Kepala Batas

Anjuran Unit Matematik
Bahagian Pendidikan Menengah



THANKS A LOT!



Do you have any questions?


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$$\begin{aligned} X^3 &= C^3 + D^1 \\ 35 &= C^4 \sin^2 \\ &= 62 \sin^4 \\ &= 57.75 \end{aligned}$$